
EE2020/EE2026

Tutorial 4

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Question #1

Tutorial 4 - Questions (Part 1)

Boolean algebra and minimization

1. A switching circuit has four inputs as shown. A and B represent the first and second bits of a binary number N_1 . C and D represent the first and second bits of a binary number N_2 . The output is to be 1 only if the product $N_1 \times N_2$ is less than or equal to 2.
 - (a) Write the truth table for the system.
 - (b) Write the canonical SOP and POS expressions for F .
 - (c) Draw a Karnaugh-map (K-map) for the function F .
 - (d) From the K-map, derive a simplified SOP expression for F . (*Hint: Use groups of only size 4.*)

Question #1a

Tutorial 4 - Questions (Part 1)

Boolean algebra and minimization

1. A switching circuit has four inputs as shown. A and B represent the first and second bits of a binary number N_1 . C and D represent the first and second bits of a binary number N_2 . The output is to be 1 only if the product $N_1 \times N_2$ is less than or equal to 2.
 - (a) Write the truth table for the system.
 - (b) Write the canonical SOP and POS expressions for F .
 - (c) Draw a Karnaugh-map (K-map) for the function F .
 - (d) From the K-map, derive a simplified SOP expression for F . (*Hint: Use groups of only size 4.*)

Question #1a

A	B	C	D	F
0	0	0	0	
0	0	0	1	
0	0	1	0	
0	0	1	1	
0	1	0	0	
0	1	0	1	
0	1	1	0	
0	1	1	1	
1	0	0	0	
1	0	0	1	
1	0	1	0	
1	0	1	1	
1	1	0	0	
1	1	0	1	
1	1	1	0	
1	1	1	1	



which pairs of numbers have a product greater than 2?

Question #1a

A	B	C	D	F
0	0	0	0	
0	0	0	1	
0	0	1	0	
0	0	1	1	
0	1	0	0	
0	1	0	1	
0	1	1	0	
0	1	1	1	
1	0	0	0	
1	0	0	1	
1	0	1	0	
1	0	1	1	
1	1	0	0	
1	1	0	1	
1	1	1	0	
1	1	1	1	

1 X 3

2 X 2

2 X 3

3 X 1

3 X 2

3 X 3

Question #1a

A	B	C	D	F
0	0	0	0	
0	0	0	1	
0	0	1	0	
0	0	1	1	
0	1	0	0	
0	1	0	1	
0	1	1	0	
0	1	1	1	0
1	0	0	0	
1	0	0	1	
1	0	1	0	0
1	0	1	1	0
1	1	0	0	
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

Question #1a: Answer

A	B	C	D	F
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

Question #1b

Tutorial 4 - Questions (Part 1)

Boolean algebra and minimization

1. A switching circuit has four inputs as shown. A and B represent the first and second bits of a binary number N_1 . C and D represent the first and second bits of a binary number N_2 . The output is to be 1 only if the product $N_1 \times N_2$ is less than or equal to 2.
 - (a) Write the truth table for the system.
 - (b) Write the canonical SOP and POS expressions for F .
 - (c) Draw a Karnaugh-map (K-map) for the function F .
 - (d) From the K-map, derive a simplified SOP expression for F . (*Hint: Use groups of only size 4.*)

Question #1b: SOP

A	B	C	D	F
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0



Sum Of Products:
OR minterms associated with $F=1$

Question #1b: SOP

A	B	C	D	F	
0	0	0	0	1	$\bar{A} \cdot \bar{B} \cdot \bar{C} \cdot \bar{D}$
0	0	0	1	1	$\bar{A} \cdot \bar{B} \cdot \bar{C} \cdot D$
0	0	1	0	1	$\bar{A} \cdot \bar{B} \cdot C \cdot \bar{D}$
0	0	1	1	1	$\bar{A} \cdot \bar{B} \cdot C \cdot D$
0	1	0	0	1	$\bar{A} \cdot B \cdot \bar{C} \cdot \bar{D}$
0	1	0	1	1	$\bar{A} \cdot B \cdot \bar{C} \cdot D$
0	1	1	0	1	$\bar{A} \cdot B \cdot C \cdot \bar{D}$
0	1	1	1	0	
1	0	0	0	1	$A \cdot \bar{B} \cdot \bar{C} \cdot \bar{D}$
1	0	0	1	1	$A \cdot \bar{B} \cdot \bar{C} \cdot D$
1	0	1	0	0	
1	0	1	1	0	
1	1	0	0	1	$A \cdot B \cdot \bar{C} \cdot \bar{D}$
1	1	0	1	0	
1	1	1	0	0	
1	1	1	1	0	

Question #1b: SOP Answer

A	B	C	D	F
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

$$\bar{A} \cdot \bar{B} \cdot \bar{C} \cdot \bar{D}$$

$$\bar{A} \cdot \bar{B} \cdot \bar{C} \cdot D$$

$$\bar{A} \cdot \bar{B} \cdot C \cdot \bar{D}$$

$$\bar{A} \cdot \bar{B} \cdot C \cdot D$$

$$\bar{A} \cdot B \cdot \bar{C} \cdot \bar{D}$$

$$\bar{A} \cdot B \cdot \bar{C} \cdot D$$

$$\bar{A} \cdot B \cdot C \cdot \bar{D}$$

$$A \cdot \bar{B} \cdot \bar{C} \cdot \bar{D}$$

$$A \cdot \bar{B} \cdot \bar{C} \cdot D$$

$$A \cdot B \cdot \bar{C} \cdot \bar{D}$$

$$F = \bar{A} \cdot \bar{B} \cdot \bar{C} \cdot \bar{D} + \bar{A} \cdot \bar{B} \cdot \bar{C} \cdot D + \bar{A} \cdot \bar{B} \cdot C \cdot \bar{D} + \bar{A} \cdot \bar{B} \cdot C \cdot D + \bar{A} \cdot B \cdot \bar{C} \cdot \bar{D} + \bar{A} \cdot B \cdot \bar{C} \cdot D + \bar{A} \cdot B \cdot C \cdot \bar{D} + \bar{A} \cdot B \cdot C \cdot D + A \cdot \bar{B} \cdot \bar{C} \cdot \bar{D} + A \cdot \bar{B} \cdot \bar{C} \cdot D + A \cdot \bar{B} \cdot C \cdot \bar{D} + A \cdot \bar{B} \cdot C \cdot D + A \cdot B \cdot \bar{C} \cdot \bar{D} + A \cdot B \cdot \bar{C} \cdot D + A \cdot B \cdot C \cdot \bar{D} + A \cdot B \cdot C \cdot D$$

Question #1b: POS

A	B	C	D	F
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0



Product of Sums:

AND maxterms associated with $F=0$

Question #1b: POS

A	B	C	D	F
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

$$A + \bar{B} + \bar{C} + \bar{D}$$

$$\bar{A} + B + \bar{C} + D$$

$$\bar{A} + B + \bar{C} + \bar{D}$$

$$\bar{A} + \bar{B} + C + \bar{D}$$

$$\bar{A} + \bar{B} + \bar{C} + D$$

$$\bar{A} + \bar{B} + \bar{C} + \bar{D}$$

Question #1b: POS Answer

A	B	C	D	F
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

$$A + \bar{B} + \bar{C} + \bar{D}$$

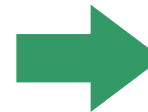
$$\bar{A} + B + \bar{C} + D$$

$$\bar{A} + B + \bar{C} + \bar{D}$$

$$\bar{A} + \bar{B} + C + \bar{D}$$

$$\bar{A} + \bar{B} + \bar{C} + D$$

$$\bar{A} + \bar{B} + \bar{C} + \bar{D}$$



$$\begin{aligned}
 F = & (A + \bar{B} + \bar{C} + \bar{D}) \cdot \\
 & (\bar{A} + B + \bar{C} + D) \cdot \\
 & (\bar{A} + B + \bar{C} + \bar{D}) \cdot \\
 & (\bar{A} + \bar{B} + C + \bar{D}) \cdot \\
 & (\bar{A} + \bar{B} + \bar{C} + D) \cdot \\
 & (\bar{A} + \bar{B} + \bar{C} + \bar{D})
 \end{aligned}$$

Question #1c

Tutorial 4 - Questions (Part 1)

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 - (b) Write the canonical SOP and POS expressions for F .
 - (c) Draw a Karnaugh-map (K-map) for the function F .
 - (d) From the K-map, derive a simplified SOP expression for F . (*Hint: Use groups of only size 4.*)

Question #1c: SOP Answer

A	B	C	D	F	
0	0	0	0	1	$\bar{A} \cdot \bar{B} \cdot \bar{C} \cdot \bar{D}$
0	0	0	1	1	$\bar{A} \cdot \bar{B} \cdot \bar{C} \cdot D$
0	0	1	0	1	$\bar{A} \cdot \bar{B} \cdot C \cdot \bar{D}$
0	0	1	1	1	$\bar{A} \cdot \bar{B} \cdot C \cdot D$
0	1	0	0	1	$\bar{A} \cdot B \cdot \bar{C} \cdot \bar{D}$
0	1	0	1	1	$\bar{A} \cdot B \cdot \bar{C} \cdot D$
0	1	1	0	1	$\bar{A} \cdot B \cdot C \cdot \bar{D}$
0	1	1	1	0	
1	0	0	0	1	$A \cdot \bar{B} \cdot \bar{C} \cdot \bar{D}$
1	0	0	1	1	$A \cdot \bar{B} \cdot \bar{C} \cdot D$
1	0	1	0	0	
1	0	1	1	0	
1	1	0	0	1	$A \cdot B \cdot \bar{C} \cdot \bar{D}$
1	1	0	1	0	
1	1	1	0	0	
1	1	1	1	0	

AB \ CD	00	01	11	10
00	1	1	1	1
01	1	1	0	1
11	1	0	0	0
10	1	1	0	0

Question #1d

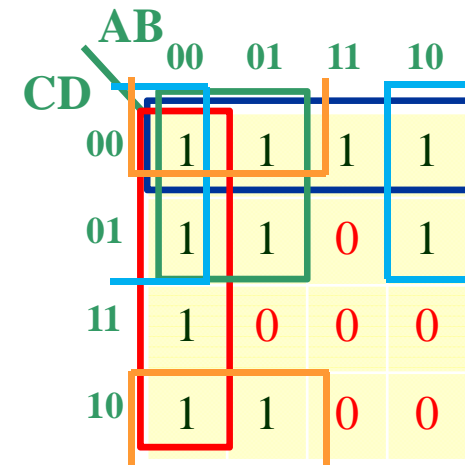
Tutorial 4 - Questions (Part 1)

Boolean algebra and minimization

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 - (d) From the K-map, derive a simplified SOP expression for F . *(Hint: Use groups of only size 4.)*

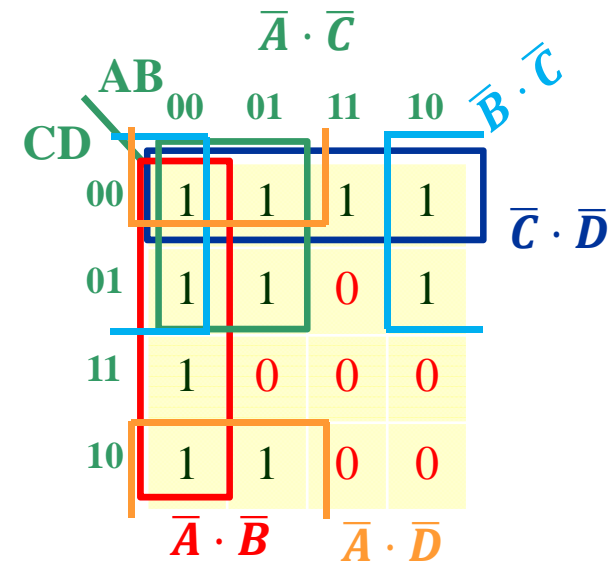
Question #1d: Minimization SOP

A	B	C	D	F	
0	0	0	0	1	$\bar{A} \cdot \bar{B} \cdot \bar{C} \cdot \bar{D}$
0	0	0	1	1	$\bar{A} \cdot \bar{B} \cdot \bar{C} \cdot D$
0	0	1	0	1	$\bar{A} \cdot \bar{B} \cdot C \cdot \bar{D}$
0	0	1	1	1	$\bar{A} \cdot \bar{B} \cdot C \cdot D$
0	1	0	0	1	$\bar{A} \cdot B \cdot \bar{C} \cdot \bar{D}$
0	1	0	1	1	$\bar{A} \cdot B \cdot \bar{C} \cdot D$
0	1	1	0	1	$\bar{A} \cdot B \cdot C \cdot \bar{D}$
0	1	1	1	0	
1	0	0	0	1	$A \cdot \bar{B} \cdot \bar{C} \cdot \bar{D}$
1	0	0	1	1	$A \cdot \bar{B} \cdot \bar{C} \cdot D$
1	0	1	0	0	
1	0	1	1	0	
1	1	0	0	1	$A \cdot B \cdot \bar{C} \cdot \bar{D}$
1	1	0	1	0	
1	1	1	0	0	
1	1	1	1	0	



Question #1d: Answer

A	B	C	D	F	
0	0	0	0	1	$\bar{A} \cdot \bar{B} \cdot \bar{C} \cdot \bar{D}$
0	0	0	1	1	$\bar{A} \cdot \bar{B} \cdot \bar{C} \cdot D$
0	0	1	0	1	$\bar{A} \cdot \bar{B} \cdot C \cdot \bar{D}$
0	0	1	1	1	$\bar{A} \cdot \bar{B} \cdot C \cdot D$
0	1	0	0	1	$\bar{A} \cdot B \cdot \bar{C} \cdot \bar{D}$
0	1	0	1	1	$\bar{A} \cdot B \cdot \bar{C} \cdot D$
0	1	1	0	1	$\bar{A} \cdot B \cdot C \cdot \bar{D}$
0	1	1	1	0	
1	0	0	0	1	$A \cdot \bar{B} \cdot \bar{C} \cdot \bar{D}$
1	0	0	1	1	$A \cdot \bar{B} \cdot \bar{C} \cdot D$
1	0	1	0	0	
1	0	1	1	0	
1	1	0	0	1	$A \cdot B \cdot \bar{C} \cdot \bar{D}$
1	1	0	1	0	
1	1	1	0	0	
1	1	1	1	0	



$$F = \bar{A} \cdot \bar{C} + \bar{B} \cdot \bar{C} + \bar{C} \cdot \bar{D} + \bar{A} \cdot \bar{D} + \bar{A} \cdot \bar{B}$$

Question #2

2. A bank vault has three locks with a different key for each lock. Each key is owned by a different person. To open the door, at least two people must insert their keys into the assigned locks. The signal lines A , B and C are 1 if there is a key inserted into lock 1, 2 or 3, respectively. Write an equation for the variable Z which is 1 if and only if the door should open.
- (a) Write the truth table for the system.
 - (b) Write the canonical SOP and POS expressions for Z .
 - (c) Draw a Karnaugh-map (K-map) for the function Z .
 - (d) From the K-map, derive a simplified SOP expression for Z .

Question #2a

2. A bank vault has three locks with a different key for each lock. Each key is owned by a different person. To open the door, at least two people must insert their keys into the assigned locks. The signal lines A , B and C are 1 if there is a key inserted into lock 1, 2 or 3, respectively. Write an equation for the variable Z which is 1 if and only if the door should open.
- (a) Write the truth table for the system.
 - (b) Write the canonical SOP and POS expressions for Z .
 - (c) Draw a Karnaugh-map (K-map) for the function Z .
 - (d) From the K-map, derive a simplified SOP expression for Z .

Question #2a

A	B	C	F
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	



find inputs with two or more 1's

Question #2a: Answer

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Question #2b

2. A bank vault has three locks with a different key for each lock. Each key is owned by a different person. To open the door, at least two people must insert their keys into the assigned locks. The signal lines A , B and C are 1 if there is a key inserted into lock 1, 2 or 3, respectively. Write an equation for the variable Z which is 1 if and only if the door should open.
- (a) Write the truth table for the system.
 - (b) Write the canonical SOP and POS expressions for Z .
 - (c) Draw a Karnaugh-map (K-map) for the function Z .
 - (d) From the K-map, derive a simplified SOP expression for Z .

Question #2b: SOP Answer

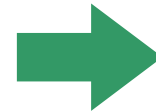
A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

$$\bar{A} \cdot B \cdot C$$

$$A \cdot \bar{B} \cdot C$$

$$A \cdot B \cdot \bar{C}$$

$$A \cdot B \cdot C$$



$$F = \bar{A} \cdot B \cdot C + A \cdot \bar{B} \cdot C + A \cdot B \cdot \bar{C} + A \cdot B \cdot C$$

Question #2b: POS Answer

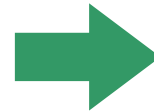
A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

$$A + B + C$$

$$A + B + \bar{C}$$

$$A + \bar{B} + C$$

$$\bar{A} + B + C$$



$$F = (A + B + C) \cdot (A + B + \bar{C}) \cdot (A + \bar{B} + C) \cdot (\bar{A} + B + C)$$

Question #2c-d

2. A bank vault has three locks with a different key for each lock. Each key is owned by a different person. To open the door, at least two people must insert their keys into the assigned locks. The signal lines A , B and C are 1 if there is a key inserted into lock 1, 2 or 3, respectively. Write an equation for the variable Z which is 1 if and only if the door should open.
- (a) Write the truth table for the system.
 - (b) Write the canonical SOP and POS expressions for Z .
 - (c) Draw a Karnaugh-map (K-map) for the function Z .
 - (d) From the K-map, derive a simplified SOP expression for Z .

Question #2c-d: Answer

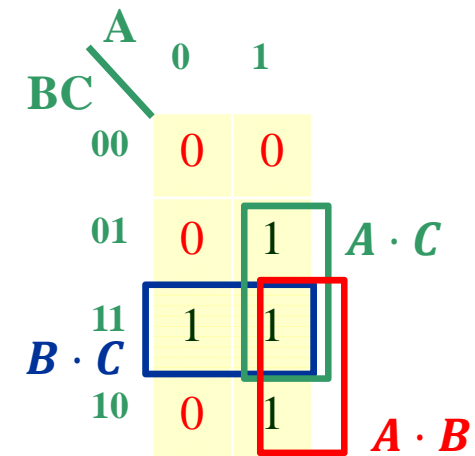
A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

$$\bar{A} \cdot B \cdot C$$

$$A \cdot \bar{B} \cdot C$$

$$A \cdot B \cdot \bar{C}$$

$$A \cdot B \cdot C$$



$$F = A \cdot C + A \cdot B + B \cdot C$$

Question #3

3. Use K-maps to obtain an MSOP and an MPOS for each of the following functions:

(a) $Z = \overline{A}\overline{B}\overline{C}D + \overline{A}B\overline{C}D + A\overline{B}\overline{C}D + \overline{A}\overline{B}CD + \overline{A}B\overline{C}\overline{D}$ with don't care for $ABCD = 1010$

(b) $Z = (\overline{A} + B + \overline{C})(A + B + \overline{C})$ with don't cares for $ABC = 111$ and 110

(c) $f(x_1, \dots, x_4) = \sum m(0, 4, 5, 6, 7) + D(1, 12, 13, 14, 15)$, where $m()$ is the set of minterms for which $f = 1$ and $D()$ is the set of don't cares. For example, $m(2)$ is the minterm corresponding to $x_1x_2x_3x_4 = 0010$ (this alternate shorthand notation is often used to express SOPs).

Question #3a

3. Use K-maps to obtain an MSOP and an MPOS for each of the following functions:

(a) $Z = \overline{A}\overline{B}\overline{C}D + \overline{A}B\overline{C}D + A\overline{B}\overline{C}D + A\overline{B}CD + A\overline{B}C\overline{D}$ with don't care for $ABCD = 1010$

(b) $Z = (A + B + C)(A + B + \overline{C})$ with don't cares for $ABC = 111$ and 110

(c) $f(x_1, \dots, x_4) = \sum m(0, 4, 5, 6, 7) + D(1, 12, 13, 14, 15)$, where $m(\)$ is the set of minterms for which $f = 1$ and $D(\)$ is the set of don't cares. For example, $m(2)$ is the minterm corresponding to $x_1x_2x_3x_4 = 0010$ (this alternate shorthand notation is often used to express SOPs).

Question #3a: SOP

$Z = \overline{A}\overline{B}\overline{C}D + \overline{A}B\overline{C}D + A\overline{B}\overline{C}D + A\overline{B}CD + A\overline{B}C\overline{D}$ with don't care for $ABCD = 1010$

CD \ AB	AB			
	00	01	11	10
00				1
01	1	1	1	1
11				
10				-

Question #3a: SOP Answer

$$Z = \overline{A}\overline{B}\overline{C}D + \overline{A}B\overline{C}D + A\overline{B}\overline{C}D + A\overline{B}CD + A\overline{B}C\overline{D} \text{ with don't care for } ABCD = 1010$$

AB \ CD	00	01	11	10	
00				1	$A \cdot \overline{B} \cdot \overline{C}$
01	1	1	1	1	$\overline{C} \cdot D$
11					
10				-	

$$F = A \cdot \overline{B} \cdot \overline{C} + \overline{C} \cdot D$$

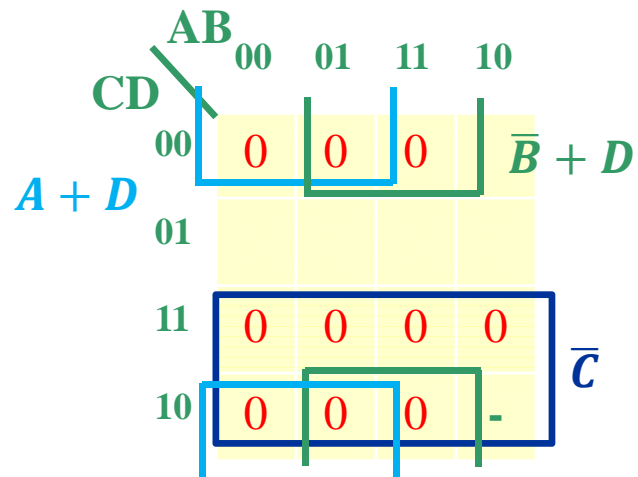
Question #3a: POS

$Z = \overline{A}\overline{B}\overline{C}D + \overline{A}B\overline{C}D + A\overline{B}\overline{C}D + A\overline{B}CD + A\overline{B}C\overline{D}$ with don't care for $ABCD = 1010$

AB \ CD		00	01	11	10
00	0	0	0	0	
01					
11	0	0	0	0	
10	0	0	0	-	

Question #3a: POS Answer

$Z = \overline{A}\overline{B}\overline{C}D + \overline{A}B\overline{C}D + A\overline{B}\overline{C}D + A\overline{B}CD + A\overline{B}C\overline{D}$ with don't care for $ABCD = 1010$



$$F = (\overline{B} + D) \cdot (A + D) \cdot \overline{C}$$

Question #3b

3. Use K-maps to obtain an MSOP and an MPOS for each of the following functions:

(a) $Z = \overline{A}\overline{B}\overline{C}D + \overline{A}B\overline{C}D + A\overline{B}\overline{C}D + \overline{A}\overline{B}CD + \overline{A}B\overline{C}\overline{D}$ with don't care for $ABCD = 1010$

(b) $Z = (\overline{A} + B + \overline{C})(A + B + \overline{C})$ with don't cares for $ABC = 111$ and 110

(c) $f(x_1, \dots, x_4) = \sum m(0, 4, 5, 6, 7) + D(1, 12, 13, 14, 15)$, where $m(\)$ is the set of minterms for which $f = 1$ and $D(\)$ is the set of don't cares. For example, $m(2)$ is the minterm corresponding to $x_1x_2x_3x_4 = 0010$ (this alternate shorthand notation is often used to express SOPs).

Question #3b: POS Answer

$Z = (\bar{A} + B + \bar{C})(A + B + \bar{C})$ with don't cares for $ABC = 111$ and 110

		A	
		0	1
BC	00		
	01	0	0
	11		-
	10		-

$B + \bar{C}$

$$F = B + \bar{C}$$

Question #3b: SOP Answer

$Z = (\bar{A} + B + \bar{C})(A + B + \bar{C})$ with don't cares for $ABC = 111$ and 110

		A		
		0	1	
BC	00	1	1	\bar{C}
	01			
	11	1	-	B
	10	1	-	

$$F = B + \bar{C}$$

Question #3c

3. Use K-maps to obtain an MSOP and an MPOS for each of the following functions:

(a) $Z = \overline{A}\overline{B}\overline{C}D + \overline{A}B\overline{C}D + A\overline{B}\overline{C}D + A\overline{B}CD + A\overline{B}C\overline{D}$ with don't care for $ABCD = 1010$

(b) $Z = (\overline{A} + B + \overline{C})(A + B + \overline{C})$ with don't cares for $ABC = 111$ and 110

(c) $f(x_1, \dots, x_4) = \sum m(0, 4, 5, 6, 7) + D(1, 12, 13, 14, 15)$, where $m(\)$ is the set of minterms for which $f = 1$ and $D(\)$ is the set of don't cares. For example, $m(2)$ is the minterm corresponding to $x_1x_2x_3x_4 = 0010$ (this alternate shorthand notation is often used to express SOPs).

Question #3c: SOP

$f(x_1, \dots, x_4) = \sum m(0, 4, 5, 6, 7) + D(1, 12, 13, 14, 15)$, where $m()$ is the set of minterms for which $f = 1$ and $D()$ is the set of don't cares. For example, $m(2)$ is the minterm corresponding to $x_1x_2x_3x_4 = 0010$ (this alternate shorthand notation is often used to express SOPs).

x_1x_2		00	01	11	10
x_3x_4	00				
	01				
	11				
	10				

Question #3c: SOP

$f(x_1, \dots, x_4) = \sum m(0, 4, 5, 6, 7) + D(1, 12, 13, 14, 15)$, where $m()$ is the set of minterms for which $f = 1$ and $D()$ is the set of don't cares. For example, $m(2)$ is the minterm corresponding to $x_1x_2x_3x_4 = 0010$ (this alternate shorthand notation is often used to express SOPs).

x_1x_2		00	01	11	10
x_3x_4	00	1			
	01				
	11				
	10				

Question #3c: SOP

$f(x_1, \dots, x_4) = \sum m(0, 4, 5, 6, 7) + D(1, 12, 13, 14, 15)$, where $m(\)$ is the set of minterms for which $f = 1$ and $D(\)$ is the set of don't cares. For example, $m(2)$ is the minterm corresponding to $x_1x_2x_3x_4 = 0010$ (this alternate shorthand notation is often used to express SOPs).

x_1x_2 x_3x_4		x_1x_2			
		00	01	11	10
00	1	1			
01					
11					
10					

Question #3c: SOP

$f(x_1, \dots, x_4) = \sum m(0, 4, 5, 6, 7) + D(1, 12, 13, 14, 15)$, where $m()$ is the set of minterms for which $f = 1$ and $D()$ is the set of don't cares. For example, $m(2)$ is the minterm corresponding to $x_1x_2x_3x_4 = 0010$ (this alternate shorthand notation is often used to express SOPs).

x_1x_2		00	01	11	10
x_3x_4	00	1	1	-	
	01	-	1	-	
	11		1	-	
	10		1	-	

Question #3c: SOP Answer

$f(x_1, \dots, x_4) = \sum m(0,4,5,6,7) + D(1,12,13,14,15)$, where $m()$ is the set of minterms for which $f=1$ and $D()$ is the set of don't cares. For example, $m(2)$ is the minterm corresponding to $x_1x_2x_3x_4 = 0010$ (this alternate shorthand notation is often used to express SOPs).

		x_1x_2			
		00	01	11	10
x_3x_4	00	1	1	-	
	01	-	1	-	
	11		1	-	
	10		1	-	
		x_2			

$$F = \overline{x_1} \cdot \overline{x_3} + x_2$$

Question #3c: POS

$f(x_1, \dots, x_4) = \sum m(0, 4, 5, 6, 7) + D(1, 12, 13, 14, 15)$, where $m(\)$ is the set of minterms for which $f = 1$ and $D(\)$ is the set of don't cares. For example, $m(2)$ is the minterm corresponding to $x_1x_2x_3x_4 = 0010$ (this alternate shorthand notation is often used to express SOPs).

x_1x_2		00	01	11	10
x_3x_4	00			-	0
	01	-		-	0
	11	0		-	0
	10	0		-	0

Question #3c: POS Answer

$f(x_1, \dots, x_4) = \sum m(0, 4, 5, 6, 7) + D(1, 12, 13, 14, 15)$, where $m(\)$ is the set of minterms for which $f = 1$ and $D(\)$ is the set of don't cares. For example, $m(2)$ is the minterm corresponding to $x_1x_2x_3x_4 = 0010$ (this alternate shorthand notation is often used to express SOPs).

x_1x_2					
		00	01	11	10
x_3x_4	00			-	0
	01	-		-	0
	11	0		-	0
	10	0		-	0

$\overline{x_1}$ (covering the first two columns, 00 and 01)
 $x_2 + \overline{x_3}$ (covering the first two rows, 00 and 01)

$$F = \overline{x_1} \cdot (x_2 + \overline{x_3})$$

Question #4

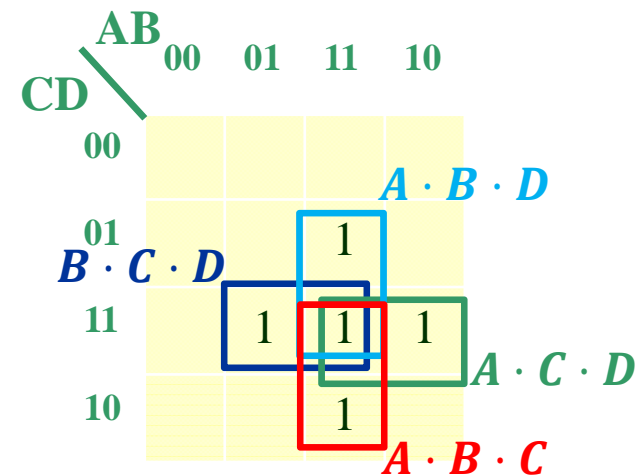
A combinational circuit has four inputs A, B, C and D and an output Z. The output is asserted whenever three or more of the inputs are asserted, otherwise the output is de-asserted. Find an MSOP expression for Z. Design combinational circuits using only 74'04 inverters, 74'00 2-input NAND gates and 74'10 3-input NAND gates. Assume that A, B and Z are active high signals, while C and D are active low signals. Use alternate gate representations for clarity of circuit diagrams.

Question #4

A	B	C	D	Z
0	0	0	0	
0	0	0	1	
0	0	1	0	
0	0	1	1	
0	1	0	0	
0	1	0	1	
0	1	1	0	
0	1	1	1	1
1	0	0	0	
1	0	0	1	
1	0	1	0	
1	0	1	1	1
1	1	0	0	
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

Question #4

A	B	C	D	Z
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1



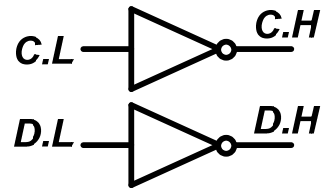
$$Z = A \cdot B \cdot D + A \cdot C \cdot D + A \cdot B \cdot C + B \cdot C \cdot D$$

Question #4

$$Z = A \cdot B \cdot D + A \cdot C \cdot D + A \cdot B \cdot C + B \cdot C \cdot D$$

A.H

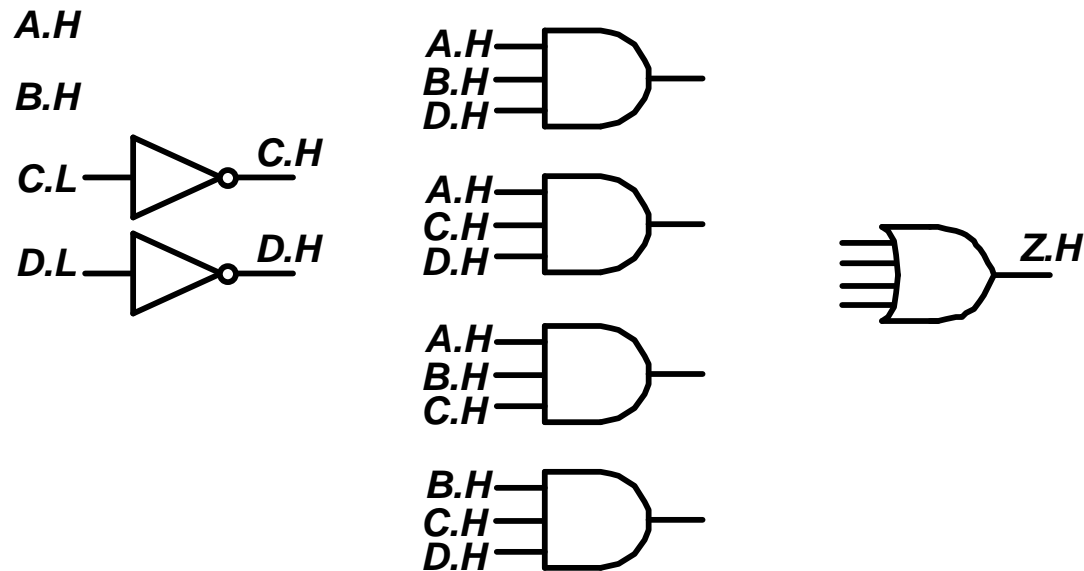
B.H



C, D are active low

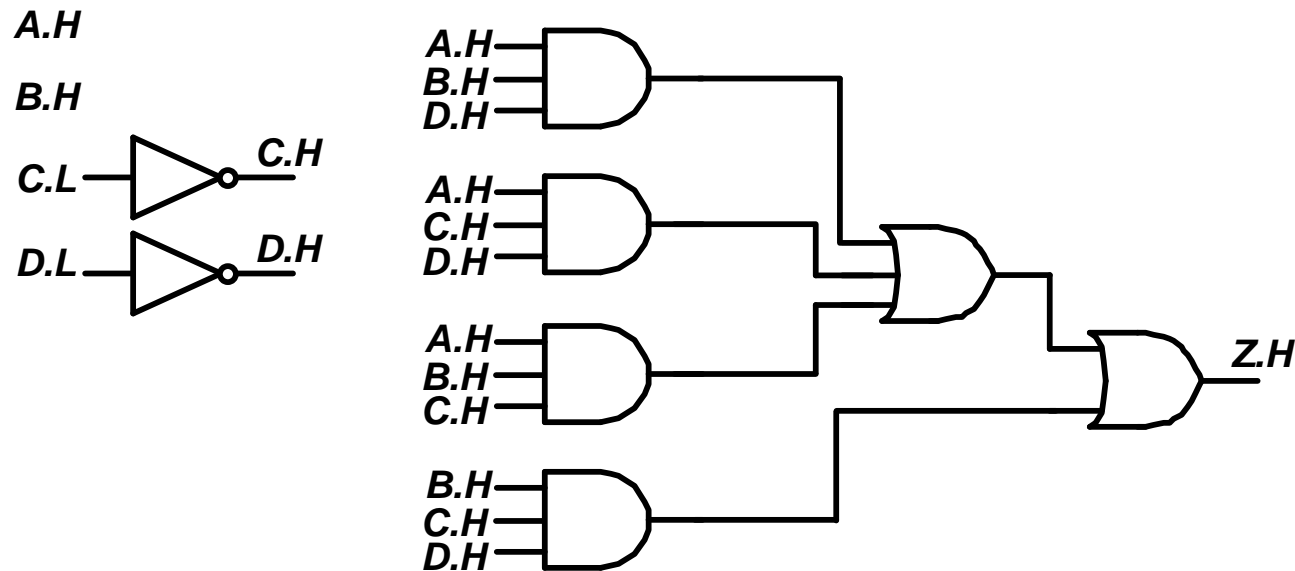
Question #4

$$Z = A \cdot B \cdot D + A \cdot C \cdot D + A \cdot B \cdot C + B \cdot C \cdot D$$



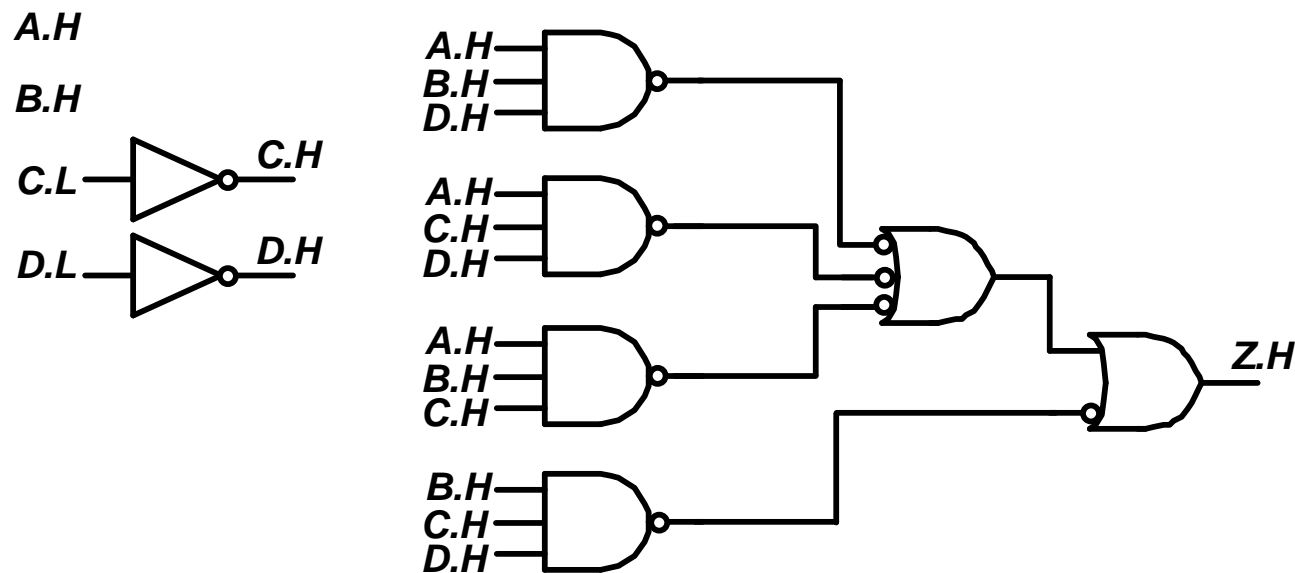
Question #4

$$Z = A \cdot B \cdot D + A \cdot C \cdot D + A \cdot B \cdot C + B \cdot C \cdot D$$



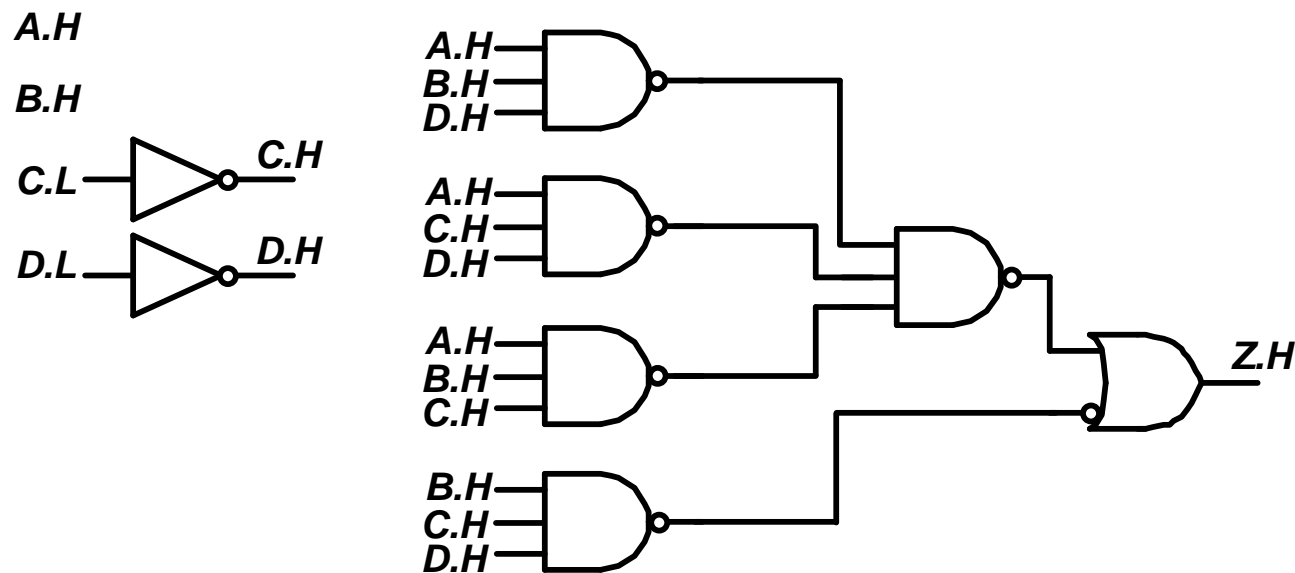
Question #4

$$Z = A \cdot B \cdot D + A \cdot C \cdot D + A \cdot B \cdot C + B \cdot C \cdot D$$



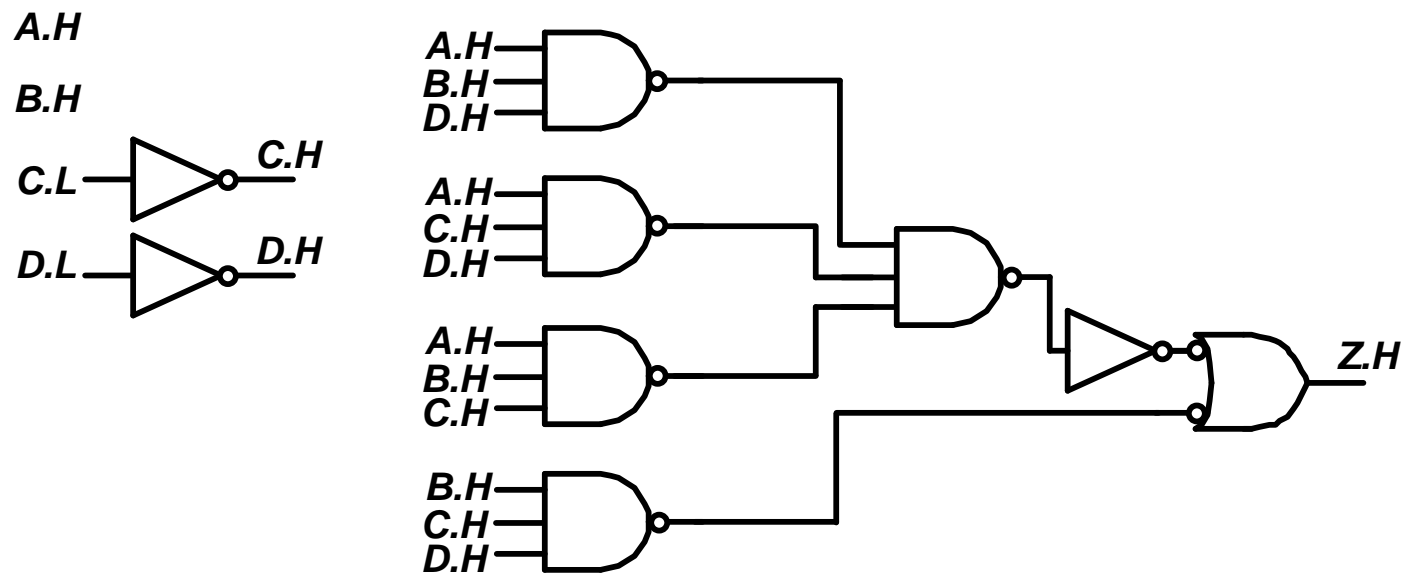
Question #4

$$Z = A \cdot B \cdot D + A \cdot C \cdot D + A \cdot B \cdot C + B \cdot C \cdot D$$



Question #4

$$Z = A \cdot B \cdot D + A \cdot C \cdot D + A \cdot B \cdot C + B \cdot C \cdot D$$



Question #4: Answer

$$Z = A \cdot B \cdot D + A \cdot C \cdot D + A \cdot B \cdot C + B \cdot C \cdot D$$

